An Experimental Study of the Dispersion in Laminar Tube Flow

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The dispersion in laminar flow through a straight tube with a circular cross section has a deep theoretical interest and considerable practical importance in many transport processes. A lot of studies have been carried out, since Taylor (1953) first presented a mathematical analysis of the convective diffusion equation to describe this problem. For a pulse or a finite slug initial input, it is interesting to note that some theoretical studies have predicted double peak behaviors in the breakthrough curves under certain conditions. This property has been confirmed experimentally, but qualitatively. However, only a little quantitative experimental study has been reported.

The representative theoretical studies have been reported by Taylor (1953), Aris (1956), Gill (1967), Gill and Ananthakrishnan (1967), Gill and Sankarasubramanian (1970), Yu (1976, 1979, 1981), Mayock et al. (1980), Andersson and Berglin (1981), Houseworth (1984), Chikwendu (1986), and Vrentas and Vrentas (1988).

These researches can be categorized according to their applicability with time and Peclet number as parameters. In general, the Taylor-Aris and Gill solutions are valid asympotically at sufficiently large values of time for arbitrary Peclet numbers. Gill et al. also provide reasonably satisfactory solutions for all the values of time at small Peclet numbers. Yu's solution, inspite of its significant numerical computation, seems to have wider applicability involving all regions of time and Peclet number. The experimental studies for the slug dispersion at the domains of low Peclet number and large dimensionless time have been performed by Bournia et al. (1961). Consequently, an attractive remainder of the Taylor diffusion problem is to make an experimental study at short times and high Peclet numbers.

The purpose of this work is to make a comprehensive experimental investigation quantitatively for a finite slug of the Basic Blue 3 dye solution dispersing in water carrier. Since the molecular diffusion coefficient of Basic Blue 3 in water is very small, the experiment was conducted essentially at high Peclet numbers ($Pe = 11,600 \sim 70,200$), and for short or middle tube lengths, at short dimensionless times ($\tau < 0.7$). Under such conditions, it was found that the breakthrough curves at a fixed tube length exhibit double peaks. This behavior has been predicted first by Gill and Ananthakrishnan (1967) and later by Golay and Atwood (1979), Mayock et al. (1980), Vanderslice et al. (1981), and Yu (1981). It has also been confirmed experimentally, but qualitatively, by Caro (1966), Golay and Atwood

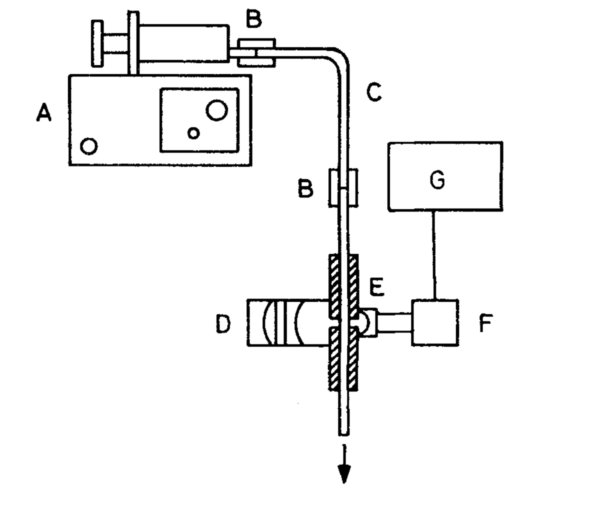

Figure 1. Experimental system of flow and measurement.

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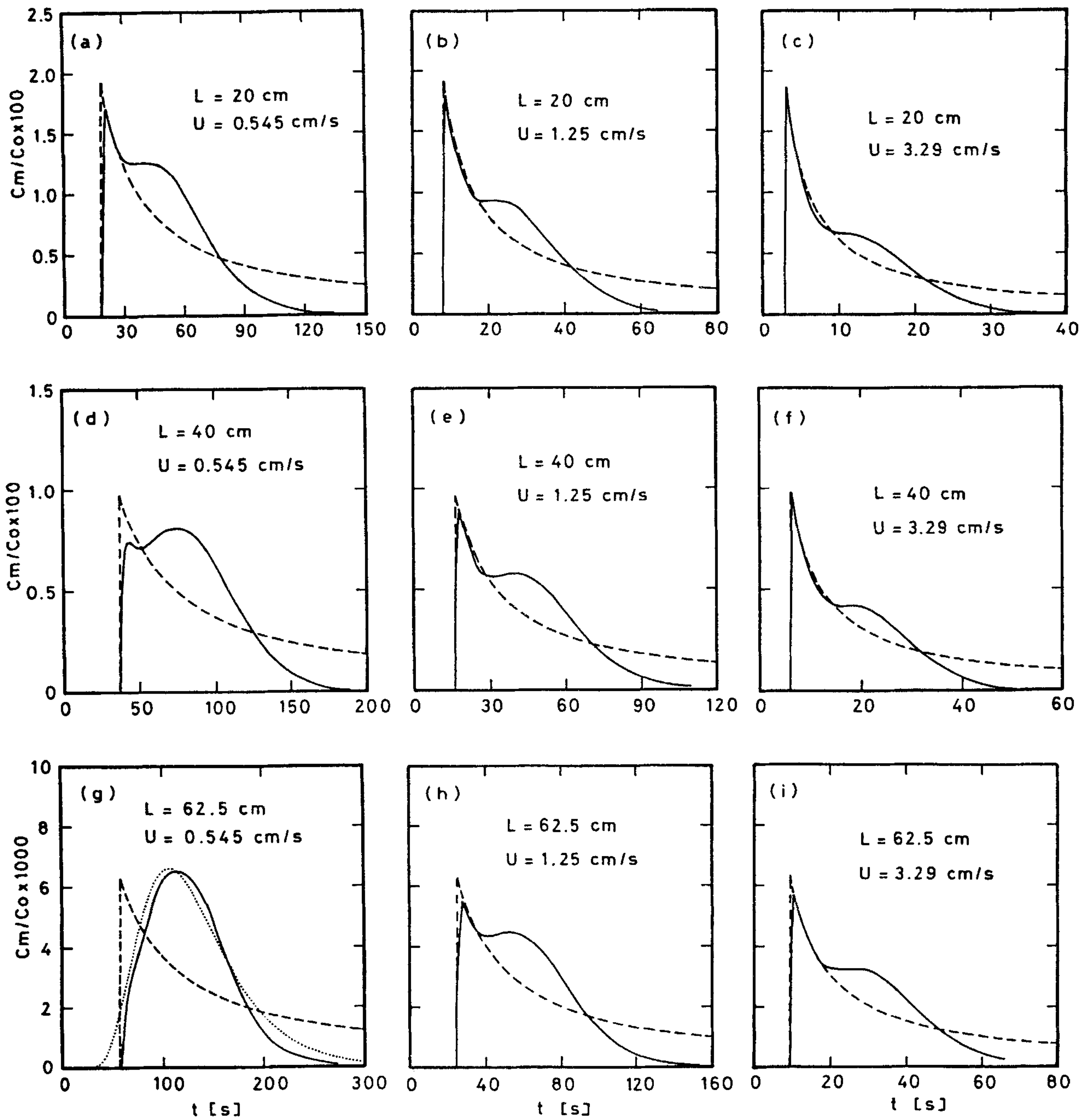

Figure 2. Breakthrough curves for various *U* and *L*.

——, experiment
------, convection
........, Taylor-Aris solution

(1979), Trumbore et al. (1985), and the authors (Korenaga et al., 1988). Although Vanderslice et al. (1981) have shown a quantitative double peak distribution as they have indicated, the ideal laminar flow can not essentially been obtained on their experimental apparatus.

Experimental Method

The slug dispersion experiments were carried out with a syringe pump (Furue Science, Japan) and a straight, transparent poly(tetrafluoroethylene) (PTFE) capillary tube of 0.806 mm ID and 1.5 mm OD. A schematic diagram of flow equipment is shown in Figure 1. A dilute solution of Basic Blue 3 dye and distilled water were used as slug and carrier solvent, respectively. The molecular diffusion coefficient of Basic Blue 3 dye in pure water at 20°C is 3.78×10^{-6} cm^2/s according to McKay et al. (1981). The solute mean concentrations over a cross section were measured by a hand-made photometric detection system at downstream. The detector was constructed mainly of a M-06110-B type fluorescent lamp as a light source (NEC, Japan), a S-1033-01 model photodiode (Hamamatsu Photonics, Japan), a 757N model logarithmic amplifier (Analog Devices, U.S.A.), and a U-228 model chart recorder (Nippon Denshi Kagaku, Japan). A flexible silicone tube (B) in Figure 1 connects two sections of PTFE tube. The light source and light accepter were mounted tightly on the outside wall of PTFE tube without any disturbance to laminar flow. The photo-diode (width, 1.5 mm; length, 6 mm) was so large that all light through a cross section can be received in order to ensure that the measured concentration is really mean concentration over a cross section. In order to reduce any possible natural convection influence resulted from the density difference between solute and water, the vertical and downflow system was adopted.

The flowing velocity rate was calculated by measuring the weight of effluent fluid at tube exit. Before the experiment was carried out the mean concentration calibration curve had been measured by passing a Basic Blue 3 dye solution of known concentration through the tube. It was found that the calibration curve is nearly linear, since the slug used was much shorter than the detection distance and was diluted rapidly to a very small concentration at the detection point. The experimental procedure is as follows.

A small volume of dye solution (2 μL) was injected into the stopped-flow system from the joint B in Figure 1 with a microsyringe. Before injection, the upper tube was removed and a small amount of water in the lower tube was taken out, thus a space which is just relative to the injected sample volume was formed. The sample was injected into this space. By such an injection method, the nearly slug initial input can be realized. Then the syring pump was started immediately. In order to ensure a fully-developed flow, two tubes were rejoined after several seconds and at the same time a chart recorder started to record breakthrough curves. The flow, which was fully developed, has been checked by injecting a sample into a flowing or a stopped system. The obtained response curves from two injecting manners were not changing. The real mean concentration distribution was obtained using a calibration curve. The slug length was calculated from injected volume and the tube diameter.

Experimental Results

The representative mean concentration distribution curves are plotted against various U and L at a same injected volume of 2 μL in Figure 2. The solid lines are experimental curves. The concentration distributions deduced by pure convection are also calculated for reference with broken lines.

The response curves in Figure 2 exhibit double peaks in all cases except for g, since the Peclet numbers used in our experiments are quite high. The high Peclet number means that the convection has a large contribution to the solute dispersion for such a long time. The double peaks have been predicted theoretically, but they were not observed in the slug dispersion experiments made by Bournia et al. (1961). This may be so, because the gases with large molecular diffusion coefficient were used or the tube length, at which the breakthrough curves were measured, was too large in their experiments. The double peaks are the combined effect of the convection and the molecular diffusion. The first peak indicates that the parabolic slug zone front has not yet been destroyed completely by molecular diffusion occurring in radial and axial directions. The second peak results from such a mechanism that the dye molecules at the front of the parabolic zone diffuse to the low velocity region near the tube wall and those at the back diffuse to the high velocity region. Therefore, the longer the residence time, the relatively higher the second peak becomes as can be seen from Figure 2. In the case of Figure 2g, which shows the longest residence time, the dispersion is controlled predominately by diffusion and tends to obey the Taylor-Aris approximation as is shown by the dot line.

In addition, by comparing Figure 2a with e and Figure 2b with i, it is found that the curve shapes are analogous when the residence time, which is defined as L/U here, has an equal or approximate value.

Acknowledgment

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Notation

a = tube radius, cm
C_0 = initial concentration
C_m = average concentration over a cross section
D = molecular diffusion coefficient, cm^2/s
L = axial distance between the middle of the injected slug and the middle of the photodiode, cm
Pe = Peclet number, $2\,aU/D$
t = time, s
U = mean flow rate over a cross section, cm/s

Greek letters

τ = dimensionless time, Dt/a^2

Literature Cited

Andersson, B., and T. Berglin, "Dispersion in Laminar Flow Through a Circular Tube," *Proc. Roy. Soc.*, **377A**, 251 (1981).
Aris, R., "On the Dispersion of a Solute in a Fluid Flowing through a Tube," *Proc. Roy. Soc.*, **235A**, 67 (1956).
Bournia, A., J. Coull, and G. Houghton, "Dispersion of Gases in Laminar Flow through a Circular Tube," *Proc. Roy. Soc.*, **261A**, 227 (1961).
Caro, C. G., "The Dispersion of Indicator Flowing through Simplified Models of the Circulation and Its Relevance to Velocity Profile in Blood Vessels," *J. Physiol.*, **185**, 501 (1966).
Chikwendu, S. C., "Application of a Slow-Zone Model to Contaminant Dispersion in Laminar Shear Flows," *Int. J. Eng. Sci.*, **24**, 1031 (1986).
Gill, W. N., "A Note on the Solution of Transient Dispersion Problems," *Proc. Roy. Soc.*, **298A**, 335 (1967).
Gill, W. N., and V. Ananthakrishnan, "Laminar Dispersion in Capillaries: Part IV. The Slug Stimulus," *AIChE J.*, **13**, 801 (1967).
Gill, W. N., and R. Sankarasubramanian, "Exact Analysis of Unsteady Convection Diffusion," *Proc. Roy. Soc.*, **316A**, 341 (1970).
Golay, M. J. E. and J. G. Atwood, "Early Phases of the Dispersion of a Sample Injected in Poiseuille Flow," *J. Chromato.*, **186**, 353 (1979).
Houseworth, J. E., "Shear Dispersion and Residence Time for Laminar Flow in Capillary Tubes," *J. Fluid Meth.*, **142**, 289 (1984).
Korenaga, T., F. Shen, H. Yoshida, and T. Takahashi, "Study of Sample Dispersion in Flow Injection Analysis," *Anal. Chim. Acta.* **214**, 97 (1988).
Mayock, K. P., J. M. Tarbell, and J. L. Duda, "Numerical Simulation of Solute Dispersion in Laminar Tube Flow," *Sep. Sci. Technol.*, **15**, 1285 (1980).
McKay, G., M. S. Otterburn, and A. G. Sweeney, "Surface Mass Transfer Processes during Colour Removal from Effluent Using Silica," *Water Res.*, **15**, 327 (1981).
Smith, R., "Diffusion in Shear Made Easy: the Taylor Limit," *J. Fluid Mech.*, **175**, 201 (1987).
Taylor, G. I., "Dispersion of Soluble Matter in Solvent Flowing Slowly through a Tube," *Proc. Roy. Soc.*, **219A**, 186 (1953).
Trumbore, C. N., M. Grehlinger, M. Stowe, and F. M. Kelleher, "Further Experiments on a New, Fast Method for Determining Molecular Weights of Diffusing Species in a Liquid Phase," *J. Chromat.*, **322**, 443 (1985).

Vanderslice, J. T., K. K. Stewart, A. G. Rosenfeld, and D. J. Higgs, "Laminar Dispersion in Flow-injection Analysis," *Talanta*, **28**, 11 (1981).
Vrentas, J. S., and C. M. Vrentas, "Dispersion in Laminar Tube Flow at Low Peclet Numbers or Short Times," *AIChE J.*, **34**, 1423 (1988).
Yu, J. S., "An Approximate Analysis of Laminar Dispersion in Circular Tubes," *J. Appl. Mech.*, **43**, 537 (1976).
———, "On Laminar Dispersion for Flow through Round Tubes," *J. Appl. Mech.*, **46**, 750 (1979).
———, "Dispersion in Laminar Flow through Tubes by Simultaneous Diffusion and Convection," *J. Appl. Mech.*, **48**, 217 (1981).

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ERRATA

In the paper entitled "Reactions in Continuous Mixtures" by Rutherford Aris (April 1989, **35**, p. 539):

The last line of the first paragraph on p. 539 should read: ". . . Chou and Ho, 1989; Prasad et al., 1986a, b; Wittmann and Syambal, 1985)."

The first line of the second paragraph on p. 539 should read: "To the best of my knowledge, the work of Prasad et al. (1986) and Wittmann (1978) . . . "

The "Literature Cited" section on p. 548 should include the following items:

Wittmann, C. V., "Mathematical Models for Cone Devolatilization Producing Char, Tar, Gases," PhD Diss., University Microfilms, Ann Arbor, MI (1978).
Wittmann, C. V., and M. Syambal, "Continuous Reaction Mixture Model for Coal Liquefaction Kinetics," *I & EC Fund.*, **24**, 82 (1985).